The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date:December 17, 2005 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 3 hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	10		Differential Equation Rhythm
2	10		Differential Equation Blues
3	10		Stability in Two Domains
4	10		Convolution in the Abstract
5	10		Sampling in Continuous-Time
6	15		Digital Filter Analysis
7	15		Digital Filter Design
8	10		Amplitude Modulation
9	10		Amplitude Demodulation
Total	100		

Final Exam Problem 1. Differential Equation Rhythm. 10 points.

For a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+7\frac{d}{dt}y(t)+12y(t)=x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? 2 points.

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. 4 points.

(c) Find the zero-input response for the initial conditions $y(0^+) = 0$ and $y'(0^+) = 1$. 4 points.

Final Exam Problem 2. Differential Equation Blues. 10 points.

For a continuous-time linear time-invariant (LTI) system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 12 y(t) = x(t)$$

for $t \geq 0^-$.

(a) What is the transfer function? 2 points.

(b) What are the values of the poles and zeroes of the transfer function? 2 points.

(c) What is the region of convergence for the transfer function? 2 points.

(d) Give a formula for the step response of the system. 4 points.

Final Exam Problem 3. Stability in Two Domains. 10 points.

In this problem, the input signal is denoted by $x(t)$ and the output signal is denoted by the output signal $y(t)$.

(a) Is the system defined by $\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 12 y(t) = x(t)$ asymptotically stable, marginally stable, or unstable? Why? 3 points.

(b) Convert the differential equation in (a) into a difference equation by first substituting the approximations $\frac{d}{dt} y(t) \approx \frac{y(t) - y(t - T_s)}{T_s}$ and $\frac{d^2}{dt^2} y(t) \approx \frac{y(t) - 2y(t - T_s) + y(t - 2T_s)}{T_s^2}$ and then sampling at $t = n\, T_s$, where T_s is the sampling period. 3 points.

(c) Is the difference equation you derived in part (b) asymptotically stable, marginally stable, or unstable? Why? 4 points.

Final Exam Problem 4. Convolution in the Abstract. 10 points.

(a) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false*: The convolution of a finite duration discrete-time signal (other than a signal that is identically zero for all time) and an infinite duration discrete-time signal always produces an infinite duration discrete-time result. 5 points.

(b) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false*: The continuous-time convolution of two signals $f(t)$ and $g(t)$ can always be computed by taking the inverse Laplace transform of $F(s)\ G(s)$. You can assume that $F(s)$ and $G(s)$ exist. 5 points.

Final Exam Problem 5. Sampling in Continuous Time. 10 points.

Sampling of an analog continuous-time signal $f(t)$ can be modeled in continuous-time as

$$y(t) = f(t)\, p(t)$$

where $p(t)$ is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

such that T_s is the sampling duration. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s}\left(1 + 2\cos(\omega_s\, t) + 2\cos(2\,\omega_s\, t) + \ldots\right)$$

where $\omega_s = 2\,\pi\, /\, T_s$.

(a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. 2 points.

(b) Find $P(\omega)$, the Fourier transform of $p(t)$. 2 points.

(c) Express your answer for $P(\omega)$ in part (b) as an impulse train in the Fourier domain. 3 points.

(d) What is the spacing of the impulse train $P(\omega)$ with respect to ω? 3 points.

Final Exam Problem 6. *Digital Filter Analysis*. 15 points.

A causal discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following difference equation:

$$y[n] = 0.8\, y[n-1] + x[n] - 1.25\, x[n-1]$$

(a) Draw the block diagram for this filter. 3 points.

(b) What are the initial conditions? What values should they be assigned? 3 points.

(c) Find the equation for the transfer function in the z-domain including the region of convergence. 3 points.

(d) Find the equation for the frequency response of the filter. 3 points.

(e) Is this filter lowpass, bandpass, bandstop, highpass, notch, or allpass? Why? 3 points.

Final Exam Problem 7. *Digital Filter Design*. 15 points.

Digital Subscriber Line (DSL) systems transmit voice and data over a telephone line using frequencies from 0 Hz to 1.1 MHz. DSL systems use a sampling rate of 2.2 MHz.

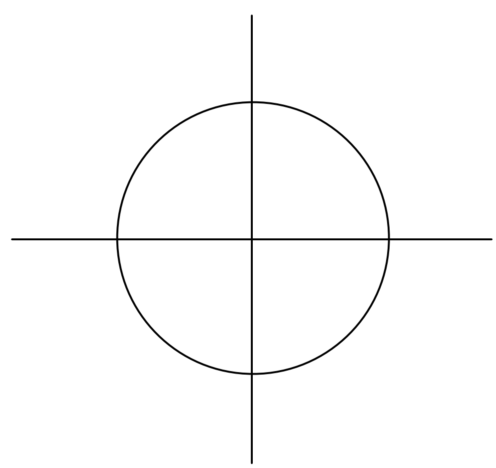
Consider an AM radio station that has a carrier frequency of 550 kHz, has a transmission bandwidth of 10 kHz, and is interfering with DSL transmission.

Design a digital filter *biquad* for the DSL receiver to reject the AM radio station but pass as much of the DSL transmission band as possible. A biquad has two poles and 0, 1, or 2 zeros.

(a) Is the digital IIR filter biquad lowpass, bandpass, bandstop, highpass, notch, or allpass? Why? 3 points.

(b) Give formulas for the locations of the poles and zeros of the biquad. 5 points.

(c) Draw the poles and zeros on the pole-zero diagram on the right. 4 points.



(d) Compute the scaling constant (gain) for the filter's transfer function. 3 points.

Final Exam Problem 8. Amplitude Modulation. 10 points.

In practice, we cannot generate a two-sided sinusoid, but we can generate a one-sided sinusoid.

Consider a one-sided cosine $c(t) = \cos(2\,\pi\, f_c\, t)\, u(t)$ where f_c is the carrier frequency (in Hz).

(a) By using the Fourier transforms of $\cos(2\,\pi\, f_c\, t)$ and $u(t)$ from a lookup table, compute the Fourier transform of $c(t) = \cos(2\,\pi\, f_c\, t)\, u(t)$ using Fourier transform properties. 3 points.

(b) Draw $|C(\omega)|$, the magnitude of the Fourier transform of $c(t)$. 3 points.

(c) Describe the differences between the magnitude of the Fourier transforms of a one-sided cosine and a two-sided cosine. What is the bandwidth of each signal? 4 points.

Lowpass
Filter

$\omega_{pass} = 2\pi f_m$
$\omega_{stop} = 2\pi (f_c - f_m)$

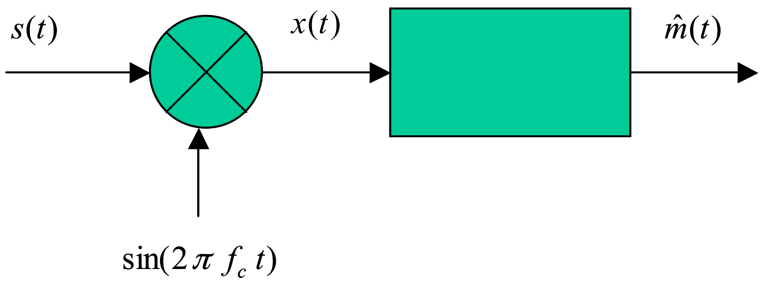
Final Exam Problem 9. Amplitude Demodulation. 10 points.

This problem explores the situation in an amplitude modulation system in which the receiver is 90 degrees out of phase with the transmitter.

A lowpass, real-valued message signal $m(t)$ with bandwidth f_m (in Hz) is to be transmitted using amplitude modulation

$$s(t) = m(t) \cos(2 \pi f_c t)$$

where f_c is the carrier frequency (in Hz) and $f_c \gg f_m$. The receiver has metaphysical knowledge of the carrier frequency, but is 90 degrees out of phase with the transmitter. The receiver processing of the transmitted signal $s(t)$ to obtain an estimate of the message signal, $\hat{m}(t)$, follows:



Hence, $x(t) = s(t) \sin(2 \pi f_c t)$.

What is the value of $\hat{m}(t)$?